

Problem 2.55

[Difficulty: 3]

2.55 SAE 10W-30 oil at 100°C is pumped through a tube $L = 10$ m long, diameter $D = 20$ mm. The applied pressure difference is $\Delta p = 5$ kPa. On the centerline of the tube is a metal filament of diameter $d = 1$ μm. The theoretical velocity profile for laminar flow through the tube is:

$$V(r) = \frac{1}{16\mu} \left(\frac{\Delta p}{L} \right) \left[d^2 - 4r^2 - \frac{D^2 - d^2}{\ln\left(\frac{d}{D}\right)} \cdot \ln\left(\frac{2r}{d}\right) \right]$$

Show that the no-slip condition is satisfied by this expression. Find the location at which the shear stress is zero, and the stress on the tube and on the filament. Plot the velocity distribution and the stress distribution. (For the stress curve, set an upper limit on stress of 5 Pa.) Discuss the results.

Given: Data on flow through a tube with a filament

Find: Whether no-slip is satisfied; location of zero stress; stress on tube and filament

Solution:

The velocity profile is

$$V(r) = \frac{1}{16\mu} \cdot \frac{\Delta p}{L} \cdot \left(d^2 - 4r^2 - \frac{D^2 - d^2}{\ln\left(\frac{d}{D}\right)} \cdot \ln\left(\frac{2r}{d}\right) \right)$$

Check the no-slip condition.
When

$$r = \frac{D}{2} \quad V\left(\frac{D}{2}\right) = \frac{1}{16\mu} \cdot \frac{\Delta p}{L} \cdot \left(d^2 - D^2 - \frac{D^2 - d^2}{\ln\left(\frac{d}{D}\right)} \cdot \ln\left(\frac{D}{d}\right) \right)$$

$$V(D) = \frac{1}{16\mu} \cdot \frac{\Delta p}{L} \cdot [d^2 - D^2 + (D^2 - d^2)] = 0$$

When

$$r = \frac{d}{2} \quad V(d) = \frac{1}{16\mu} \cdot \frac{\Delta p}{L} \cdot \left(d^2 - d^2 - \frac{D^2 - d^2}{\ln\left(\frac{d}{D}\right)} \cdot \ln\left(\frac{d}{d}\right) \right) = 0$$

The no-slip condition is satisfied.

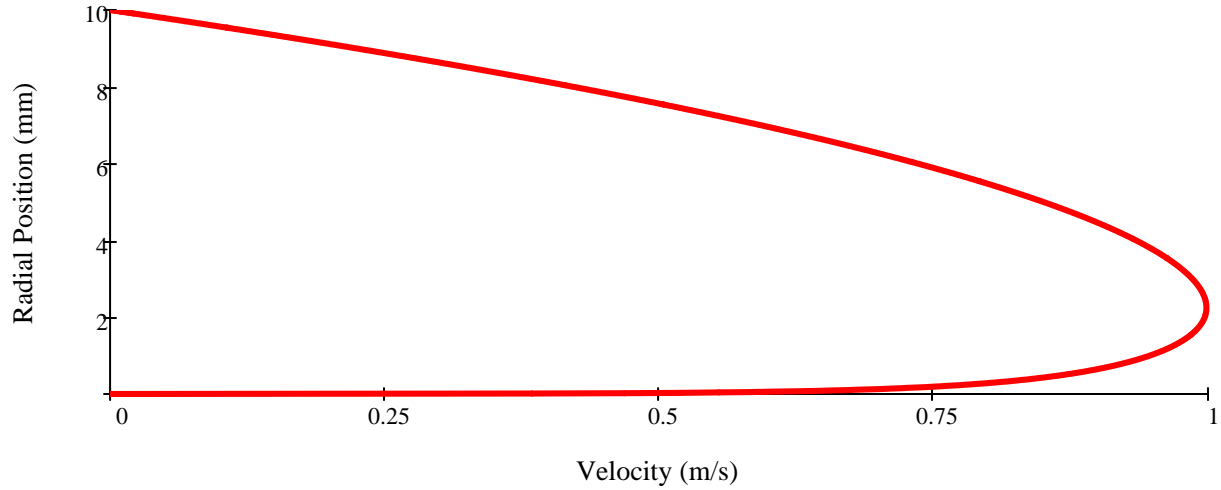
The given data is

$d = 1 \cdot \mu\text{m}$	$D = 20 \cdot \text{mm}$	$\Delta p = 5 \cdot \text{kPa}$	$L = 10 \cdot \text{m}$
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The viscosity of SAE 10-30 oil at 100°C is (Fig. A.2)

$$\mu = 1 \times 10^{-2} \cdot \frac{\text{N} \cdot \text{s}}{\text{m}^2}$$

The plot looks like



For each, shear stress is given by

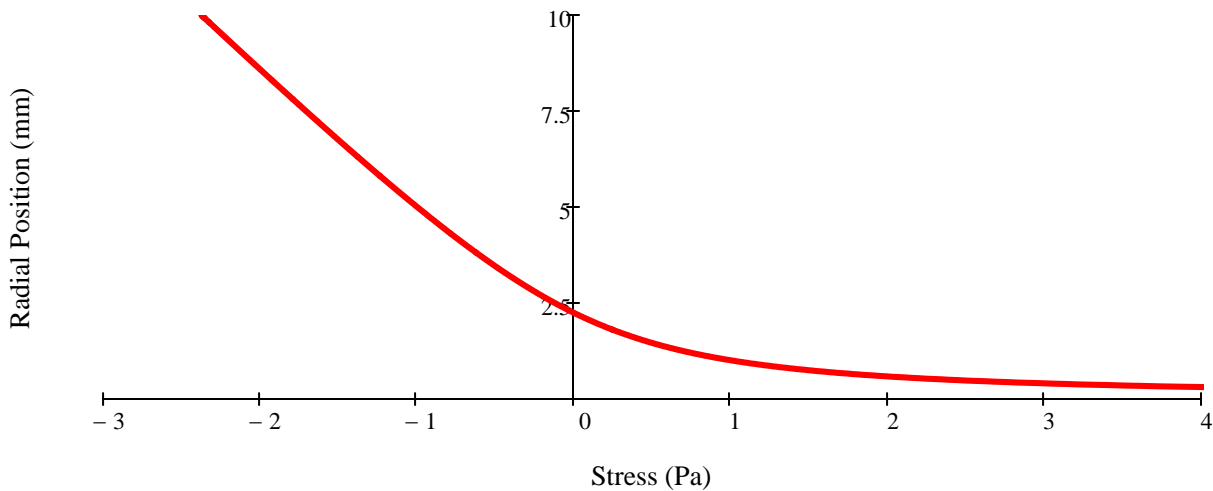
$$\tau_{rx} = \mu \cdot \frac{du}{dr}$$

$$\tau_{rx} = \mu \cdot \frac{dV(r)}{dr} = \mu \cdot \frac{d}{dr} \left[\frac{1}{16 \cdot \mu} \cdot \frac{\Delta p}{L} \cdot \left(d^2 - 4 \cdot r^2 - \frac{D^2 - d^2}{\ln\left(\frac{d}{D}\right)} \cdot \ln\left(\frac{2 \cdot r}{D_i}\right) \right) \right]$$

$$\tau_{rx}(r) = \frac{1}{16} \cdot \frac{\Delta p}{L} \cdot \left(-8 \cdot r - \frac{D^2 - d^2}{\ln\left(\frac{d}{D}\right)} \cdot r \right)$$

For the zero-stress point

$$-8 \cdot r - \frac{D^2 - d^2}{\ln\left(\frac{d}{D}\right)} \cdot r = 0 \quad \text{or} \quad r = \sqrt{\frac{d^2 - D^2}{8 \cdot \ln\left(\frac{d}{D}\right)}} \quad r = 2.25 \cdot \text{mm}$$



Using the stress formula

$$\tau_{rx}\left(\frac{D}{2}\right) = -2.374 \text{ Pa}$$

$$\tau_{rx}\left(\frac{d}{2}\right) = 2.524 \cdot \text{kPa}$$